# ChE-402: Diffusion and Mass Transfer

Lecture 8

# Intended Learning Outcome

- Analyze dimensional analysis based mass transfer correlations.
- Use mass transfer correlations to calculate mass transfer coefficient.
- Analysis of some more problems applying mass transfer correlations.
- Apply three different mass transfer theories to understand dependence on mass transfer coefficient on diffusivity and velocity.
- Understand why mass transfer theories, specially related to fluid-fluid interface in predicting the mass transfer coefficient?

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# Why we need correlation for mass transfer coefficient

- **Mass transfer coefficients, *k*, do not have molecular basis as that of diffusion coefficient, *D*.**
- **For example *k* (oxygen dissolving in water) will differ based on stirring, whereas *D* will not change.**

<u>The relevance of mass transfer coefficient in the chemical industry (process scale-up)</u>

Small-scale (10 L) bioreactor

Laboratory studies

Large-scale (10000 L) bioreactor

Manufacturing

# Dimensionless analysis

**There are several industrially important problems of interest:**

1) Fluid flow through a packed bed of particles (ion-exchange, adsorption, absorption, catalysis...).
2) Gas bubbles rising in a liquid tank (humidification, absorption, oxygenation).
3) Falling films (humidification).

- Typically, several experiments are done in a small prototype (at small scale) to correlate parameters for dimensionless correlations or validate if existing.
- Dimensionless correlations are applied for scale-up.
- The error in such correlation can be as low as 1%, and as high as 30% depending on correlation accuracy and type of interface (fluid-fluid vs fluid-solid).
- This method has been heavily inspired by heat transport analysis which has been carried out for a longer period of history for fluid-solid interfaces.

$$\frac{kl}{D} = (\text{constant})\ (\text{Re})^{x}(\text{Sc})^{y}$$

# Variables important to dimensionless analysis

$D = \text{mass diffusivity} =$

$\alpha = \text{thermal diffusivity} = \dfrac{\text{heat conducted}}{\text{heat stored}}$

$\nu = \text{momentum diffusivity} =$

$\text{Schmidt number} = \dfrac{\nu}{\text{D}}$

$\dfrac{\text{momentum}}{\text{mass}}$

$\text{Lewis number} = \dfrac{\alpha}{\text{D}}$

$\dfrac{\text{heat}}{\text{mass}}$

$\text{Prandtl number} = \dfrac{\nu}{\alpha}$

$\dfrac{\text{momentum}}{\text{heat}}$

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# What other variables that come to your mind

$k$ = mass transfer coefficient = velocity of diffusion =

$v$ = velocity =

$D$ = mass diffusivity =

**Length-scale (thickness, diameter) matters**

$$\text{Stanton number} = \frac{\mathrm{k}}{\mathrm{v}}$$

$$\frac{\text{velocity of diffusion}}{\text{velocity}}$$

$$\text{Sherwood number} = \frac{\mathrm{kl}}{\mathrm{D}}$$

$$\frac{\text{mass transfer velocity}}{\text{mass diffusivity}}$$

$$\text{Peclet number} = \frac{\mathrm{vl}}{\mathrm{D}}$$

$$\frac{\text{velocity}}{\text{mass diffusivity}}$$

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# Dimensional numbers important to mass transfer

| Group[a] | Physical meaning | Used in |
|---|---|---|
| Sherwood number $\frac{kl}{D}$ | $\frac{\text{mass transfer velocity}}{\text{diffusion velocity}}$ | Usual dependent variable |
| Stanton number $\frac{k}{v^0}$ | $\frac{\text{mass transfer velocity}}{\text{flow velocity}}$ | Occasional dependent variable |
| Schmidt number $\frac{\nu}{D}$ | $\frac{\text{diffusivity of momentum}}{\text{diffusivity of mass}}$ | Correlations of gas or liquid data |
| Lewis number $\frac{\alpha}{D}$ | $\frac{\text{diffusivity of energy}}{\text{diffusivity of mass}}$ | Simultaneous heat and mass transfer |

E. L. Cussler, Diffusion: Mass Transfer in Fluid Systems

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# Dimensional numbers important to mass transfer

| Group[a] | Physical meaning | Used in |
|---|---|---|
| Reynolds number $\frac{lv}{\nu}$ | $\frac{\text{inertial forces}}{\text{viscous forces}}$ or $\frac{\text{flow velocity}}{\text{“momentum velocity”}}$ | Forced convection |
| Grashof number $\frac{l^3 g \Delta\rho / \rho}{\nu^2}$ | $\frac{\text{buoyancy forces}}{\text{viscous forces}}$ | Free convection |
| Péclet number $\frac{v^0 l}{D}$ | $\frac{\text{flow velocity}}{\text{diffusion velocity}}$ | Correlations of gas or liquid data |
| Second Damköhler number or (Thiele modulus)$^2$ $\frac{\kappa l^2}{D}$ | $\frac{\text{reaction velocity}}{\text{diffusion velocity}}$ | Correlations involving reactions (see Chapters 16–17) |

E. L. Cussler, Diffusion: Mass Transfer in Fluid Systems

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# Frequently used mass transfer correlations

Divided into 2 categories

### Fluid-fluid interface

- Distillation
- Absorption
- Extraction
- Water aeration
- Oxygenation

Frequently used in chemical industry.
No parallel in heat transfer.

**Error of the order of 30%**

### Fluid-solid interface

- Membranes
- Adsorption
- Leaching
- Catalyst bed

Heavily inspired from heat transfer.

**Error at around 10%,**
At best 1% (laminar flow in a single tube)

Laminar flow of one fluid in a tube is much better understood than turbulent flow of gas and liquid in a packed tower

- **Due to the errors, these correlations are often used to design a small-scale (pilot plant).**
- **Typically you will validate the result in pilot plant with the actual chemicals, and then make a scale-up model.**

# Frequently used mass transfer correlations

**General patterns: correlation of *Sh (or St)* to *Re and Sc***

$$\text{Sherwood number} = \frac{\text{kl}}{\text{D}} \qquad \text{Reynolds number} = \frac{\text{dv}}{\nu}$$

$$\text{Stanton number} = \frac{\text{k}}{\text{v}} \qquad \text{Schmidt number} = \frac{\nu}{\text{D}}$$

| | | | |
|---|---|---|---|
| Packed beds | $\frac{k}{v^0} = 1.17\left(\frac{dv^0}{\nu}\right)^{-0.42}\left(\frac{D}{\nu}\right)^{2/3}$ | $d$ = particle diameter<br>$v^0$ = superficial velocity | The superficial velocity is that which would exist without packing |

**Can you write above correlation in terms of Sh (or St) as a function of Sc, and Re ??**

$$\text{St} = 1.17\ \text{Re}^{-0.42}\ \text{Sc}^{-2/3}$$

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# Fluid-fluid interface (Packed Tower)

| Physical situation | Basic equation[b] | Key variables | Remarks |
|---|---|---|---|
| Liquid in a packed tower | $k\left(\frac{1}{\nu g}\right)^{1/3} = 0.0051\left(\frac{v^0}{a\nu}\right)^{0.67}\left(\frac{D}{\nu}\right)^{0.50}(ad)^{0.4}$ | $a$ = packing area per bed volume<br>$d$ = nominal packing size | Probably the best available correlation for liquids; tends to give lower value than other correlations |
| | $\frac{kd}{D} = 25\left(\frac{dv^0}{\nu}\right)^{0.45}\left(\frac{\nu}{D}\right)^{0.5}$ | $d$ = nominal packing size | The classical result, widely quoted; probably less successful than above |
| Gas in a packed tower | $\frac{k}{aD} = 3.6\left(\frac{v^0}{a\nu}\right)^{0.70}\left(\frac{\nu}{D}\right)^{1/3}(ad)^{-2.0}$ | $a$ = packing area per bed volume<br>$d$ = nominal packing size | Probably the best available correlation for gases |
| | $\frac{kd}{D} = 1.2\,(1-\varepsilon)^{0.36}\left(\frac{dv^0}{\nu}\right)^{0.64}\left(\frac{\nu}{D}\right)^{1/3}$ | $d$ = nominal packing size<br>$\varepsilon$ = bed void fraction | Again, the most widely quoted classical result |

$k\left(\frac{1}{\nu g}\right)^{1/3}$ is unusual form of St

$v^0$ is superficial velocity (Velocity that exists without packing)

**Can you write above correlations in terms of Sh (or St) and Re and Sc ?**

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# Fluid-fluid interface (Gas bubbles in tank)

| Physical situation | Basic equation[b] | Key variables | Remarks |
|---|---|---|---|
| Pure gas bubbles in a stirred tank | $\frac{kd}{D} = 0.13\left(\frac{(P/V)\,d^4}{\rho\nu^3}\right)^{1/4}\left(\frac{\nu}{D}\right)^{1/3}$ | $d$ = bubble diameter<br>$P/V$ = stirrer power per volume | Note that $k$ does not depend on bubble size |
| Pure gas bubbles in an unstirred tank | $\frac{kd}{D} = 0.31\left(\frac{d^3\,g\,\Delta\rho/\rho}{\nu^2}\right)^{1/3}\left(\frac{\nu}{D}\right)^{1/3}$ | $d$ = bubble diameter<br>$\Delta\rho$ = density difference between bubble and surrounding fluid | 0.3-cm diameter or larger |

$\frac{d^3 g(\Delta\rho/\rho)}{\nu^2}$ is Grashof number

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# Fluid-fluid interface (some others)

| Physical situation | Basic equation[b] | Key variables | Remarks |
|---|---|---|---|
| Small liquid drops rising in unstirred solution | $\frac{kd}{D} = 1.13\left(\frac{dv^0}{D}\right)^{0.8}$ | $d$ = drop diameter<br>$v^0$ = drop velocity | These small drops behave like rigid spheres |
| Falling films | $\frac{kz}{D} = 0.69\left(\frac{zv^0}{D}\right)^{0.5}$ | $z$ = position along film<br>$v^0$ = average film velocity | Frequently embroidered and embellished |

**What dimensionless number are these ?**

$$\text{Peclet number} = \frac{\text{vl}}{\text{D}}$$

$$\frac{\text{velocity}}{\text{mass diffusivity}}$$

# Fluid-solid interface

Table 8.3-3 *Selected mass transfer correlations for fluid–solid interfaces*[a]

| Physical situation | Basic equation[b] | Key variables | Remarks |
|---|---|---|---|
| Membrane | $\frac{kl}{D} = 1$ | $l$ = membrane thickness | Often applied even where membrane is hypothetical |
| Laminar flow along flat plate[c] | $\frac{kL}{D} = 0.646 \left(\frac{Lv^0}{\nu}\right)^{1/3} \left(\frac{\nu}{D}\right)^{1/3}$ | $L$ = plate length<br>$v^0$ = bulk velocity | Solid theoretical foundation, which is unusual $Re < 2000$ |
| Turbulent flow through horizontal slit | $\frac{kd}{D} = 0.026 \left(\frac{dv^0}{\nu}\right)^{0.8} \left(\frac{\nu}{D}\right)^{1/3}$ | $v^0$ = average velocity in slit<br>$d = [2/\pi]$ (slit width) | Mass transfer here is identical with that in a pipe of equal wetted perimeter |
| Turbulent flow through circular pipe | $\frac{kd}{D} = 0.026 \left(\frac{dv^0}{\nu}\right)^{0.8} \left(\frac{\nu}{D}\right)^{1/3}$ | $v^0$ = average velocity in slit<br>$d$ = pipe diameter | Same as slit, because only wall regime is involved |
| Laminar flow through circular tube | $\frac{kd}{D} = 1.62 \left(\frac{d^2v^0}{LD}\right)^{1/3}$ | $d$ = pipe diameter<br>$L$ = pipe length<br>$v^0$ = average velocity in tube | Very strong theoretical and experimental basis $Re < 2000$ |
| Flow outside and parallel to a capillary bed | $\frac{kd}{D} = 1.25 \left(\frac{d^2v^0}{\nu l}\right)^{0.93} \left(\frac{\nu}{D}\right)^{1/3}$ | $d$ = 4 cross-sectional area/(wetted perimeter)<br>$v^0$ = superficial velocity | Not reliable because of channeling in bed |
| Flow outside and perpendicular to a capillary bed | $\frac{kd}{D} = 0.80 \left(\frac{dv^0}{\nu}\right)^{0.47} \left(\frac{\nu}{D}\right)^{1/3}$ | $d$ = capillary diameter<br>$v^0$ = velocity approaching bed | Reliable if capillaries evenly spaced |
| Forced convection around a solid sphere | $\frac{kd}{D} = 2.0 + 0.6 \left(\frac{dv^0}{\nu}\right)^{1/2} \left(\frac{\nu}{D}\right)^{1/3}$ | $d$ = sphere diameter<br>$v^0$ = velocity of sphere | Very difficult to reach $(kd/D) = 2$ experimentally; no sudden laminar-turbulent transition |
| Free convection around a solid sphere | $\frac{kd}{D} = 2.0 + 0.6 \left(\frac{d^3\Delta\rho g}{\rho\nu^2}\right)^{1/4} \left(\frac{\nu}{D}\right)^{1/3}$ | $d$ = sphere diameter<br>$g$ = gravitational acceleration | For a 1-cm sphere in water, free convection is important when $\Delta\rho = 10^{-9}$ g/cm$^3$ |
| Packed beds | $\frac{k}{v^0} = 1.17 \left(\frac{dv^0}{\nu}\right)^{-0.42} \left(\frac{D}{\nu}\right)^{2/3}$ | $d$ = particle diameter<br>$v^0$ = superficial velocity | The superficial velocity is that which would exist without packing |
| Spinning disc | $\frac{kd}{D} = 0.62 \left(\frac{d^2\omega}{\nu}\right)^{1/2} \left(\frac{\nu}{D}\right)^{1/3}$ | $d$ = disc diameter<br>$\omega$ = disc rotation (radians/time) | Valid for Reynolds numbers between 100 and 20,000 |

*Notes:* [a] The symbols used include the following: $D$ is the diffusion coefficient of the material being transferred; $k$ is the local mass transfer coefficient; $\rho$ is the fluid density; $\nu$ is the kinetmatic viscosity. Other symbols are defined for the specific situation.

[b] The dimensionless groups are defined as follows: $(dv^0/\nu)$ and $(d^2\omega/\nu)$ are the Reynolds number; $\nu/D$ is the Schmidt number; $(d^3\Delta\rho g/\rho\nu^2)$ is the Grashof number, $kd/D$ is the Sherwood number; $k/v^0$ is the Stanton number.

[c] The mass transfer coefficient given here is the value averaged over the length $L$.

# More correlations in Perry's Handbook

- Packed bed of particles (adsorption, ion-exchange, reaction)
- Packed bed of 2-phase contractors (absorption, distillation, etc.)
- Agitated systems
- Drops and bubbles
- Submerged objects
- Wetted wall columns
- Flat plates

Perry's Chemical Engineer's Handbook, 8th edition, chapter 5

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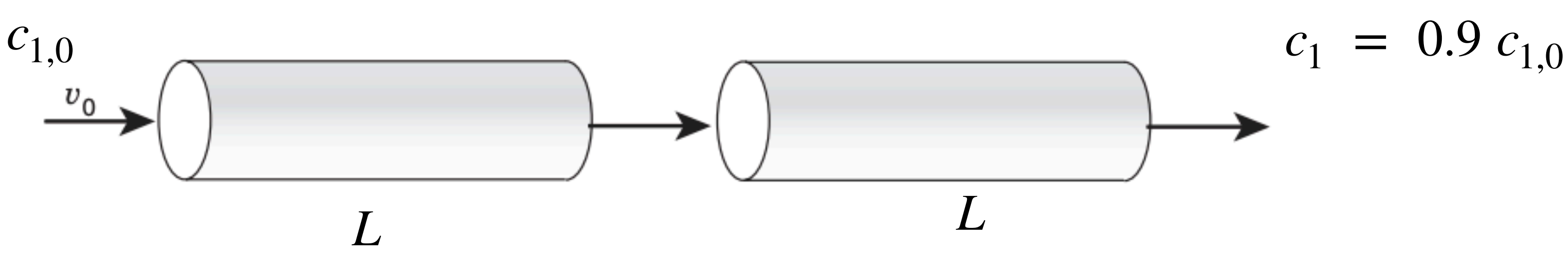
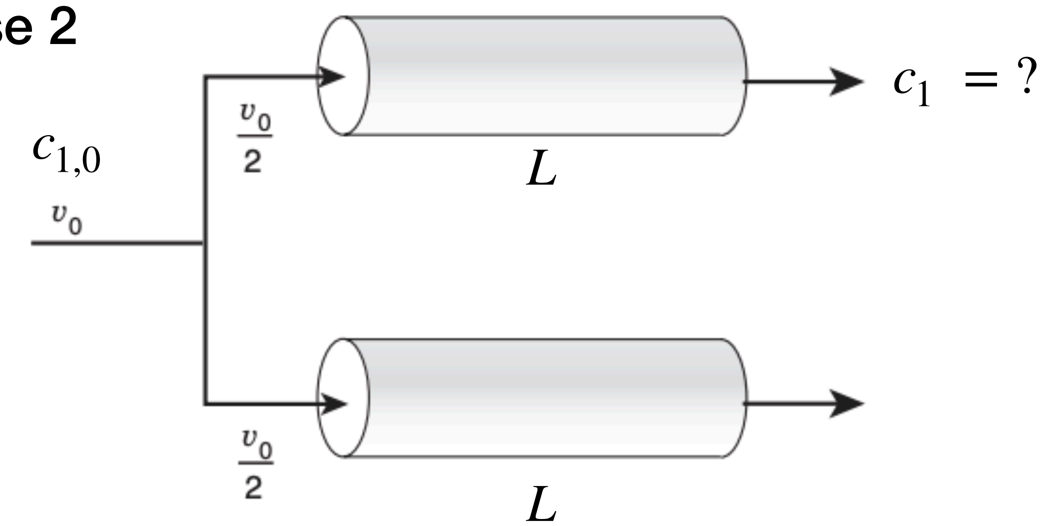
# In-class exercise problem 1

Compare the two cases with irreversible reaction in packed bed and calculate $c_1$ in case 2.

Case 1




Case 2

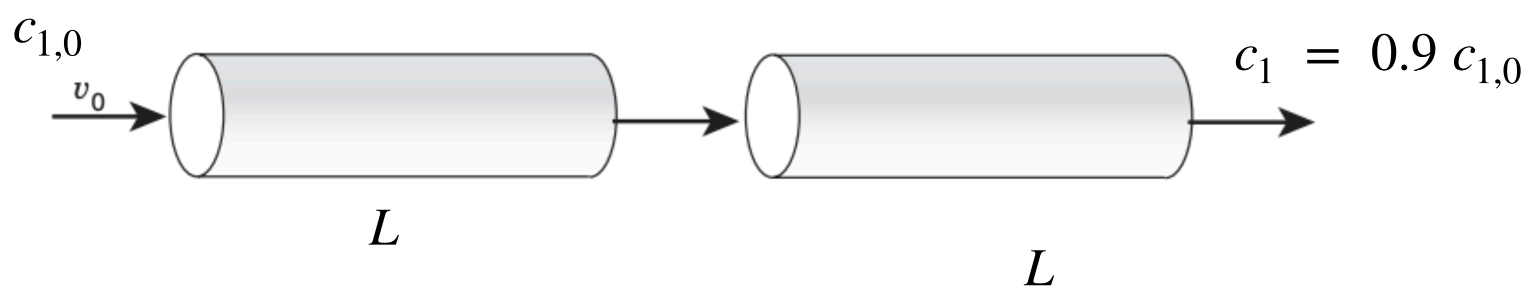



Fluid is reacting with a bed (fluid-solid correlation)

$$\frac{k}{v} = 1.17\left(\frac{dv}{\nu}\right)^{-0.42}\left(\frac{D}{\nu}\right)^{2/3}$$

$$c_1 = c_{1,0}\exp\left(-\frac{kaz}{v}\right)$$

## Case 1




$$\frac{k}{v^0} = 1.17 \left(\frac{dv^0}{\nu}\right)^{-0.42} \left(\frac{D}{\nu}\right)^{2/3}$$

$$c_1 = c_{1,0} \exp\left(-\frac{kaz}{v^0}\right)$$

$$z = 2L$$

$$k = constant * (v^0)^{0.58}$$

$$c_1 = c_{1,0} \exp\left(-\frac{constant * (v^0)^{0.58} a(2L)}{v^0}\right)$$

$$c_1 = c_{1,0} \exp\left(-\frac{2 * constant * aL}{(v^0)^{0.42}}\right) = 0.9 c_{1,0}$$

$$\Rightarrow \frac{constant * aL}{(v^0)^{0.42}} = -\frac{\ln(0.9)}{2}$$

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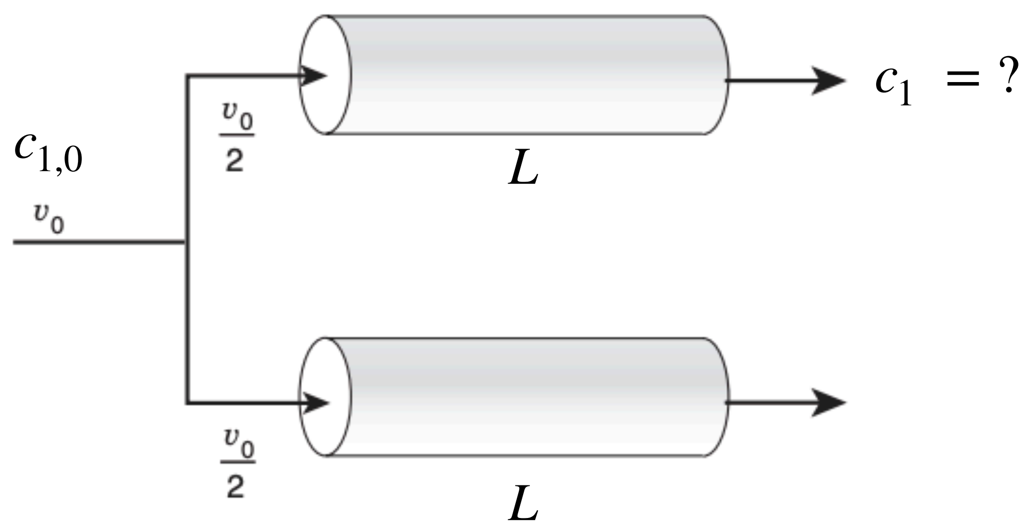
## Case 2




$$c_1 = c_{1,0} \exp\left(-\frac{kaz}{(v^0/2)}\right) \qquad z = L$$

$$\Rightarrow c_1 = c_{1,0} \exp\left(-\frac{constant * (v^0/2)^{0.58} aL}{v^0/2}\right)$$

$$\Rightarrow c_1 = c_{1,0} \exp\left(-\frac{constant * aL}{(v^0/2)^{0.42}}\right)$$

$$\frac{k}{v^0} = 1.17\left(\frac{dv^0}{\nu}\right)^{-0.42}\left(\frac{D}{\nu}\right)^{2/3}$$

$$\Rightarrow c_1 = c_{1,0} \exp\left(-\frac{2^{0.42} constant * aL}{(v^0)^{0.42}}\right)$$

$$k = constant * (v^0/2)^{0.58}$$

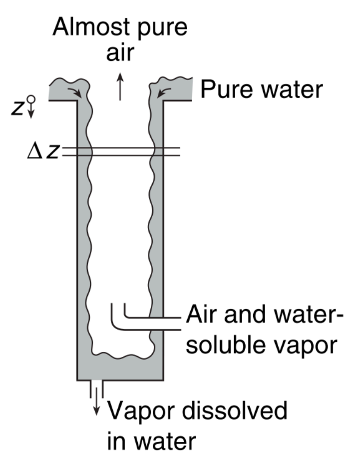
$$\frac{constant * aL}{(v^0)^{0.42}} = -\frac{\ln(0.9)}{2}$$

$$\Rightarrow c_1 = c_{1,0} \exp(2^{0.42} * \ln(0.9)/2) = 0.93 c_{1,0}$$

# Can we derive correlations from first-principle?

**Fluid-fluid interface**

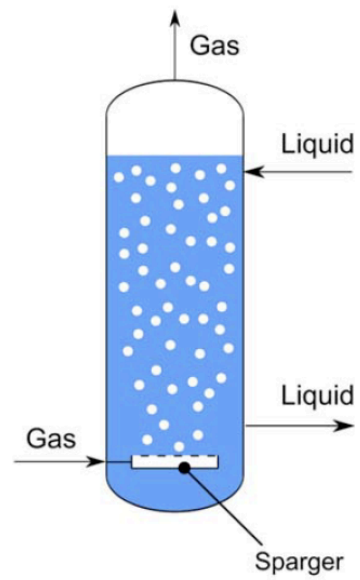
### Falling film




$$\frac{kz}{D} = 0.69\left(\frac{zv}{D}\right)^{0.5}$$

$$k \sim D^{0.5}, \quad \sim v^{0.5}$$

### Gas bubbles in tank




Source: Wikipedia, "Blasensäule"

$$\frac{kd}{D} = 0.31\left(\frac{d^3 g\Delta\rho}{\rho\nu^2}\right)^{1/3}\left(\frac{\nu}{D}\right)^{1/3}$$

$$k \sim D^{2/3}$$

### Liquid in packed tower

Source: Büchi Glas, Uster

$$k\left(\frac{1}{\nu g}\right)^{1/3} = 0.0051\left(\frac{v^0}{a\nu}\right)^{0.67}\left(\frac{D}{\nu}\right)^{0.5}(ad)^{0.4}$$

$$k \sim D^{0.5}, \quad \sim v^{0.67}$$

**Goal: can we develop correlations from first principle which describes the following relationships**

$$k \sim D^{0.5} \text{ or } D^{0.67}, \quad \sim v^{0.5} \text{ or } v^{0.67}$$

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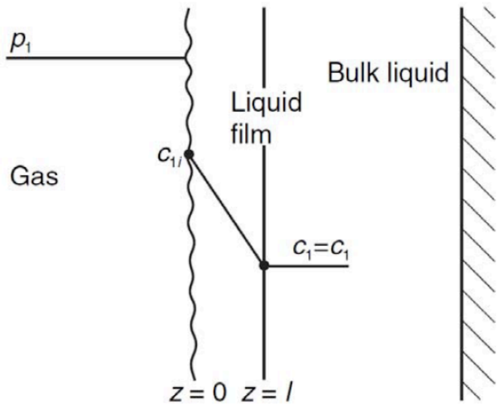
# Three prominent mass transfer coefficient theories

**Film theory**

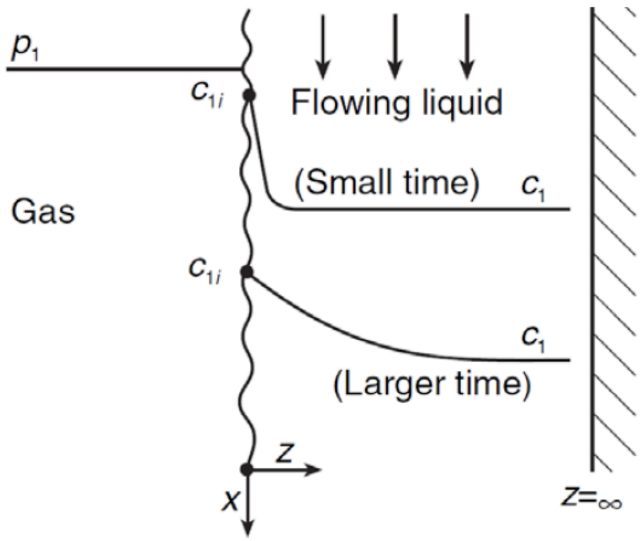
thin film diffusion




$$\frac{kl}{D} = 1$$

**Penetration theory**

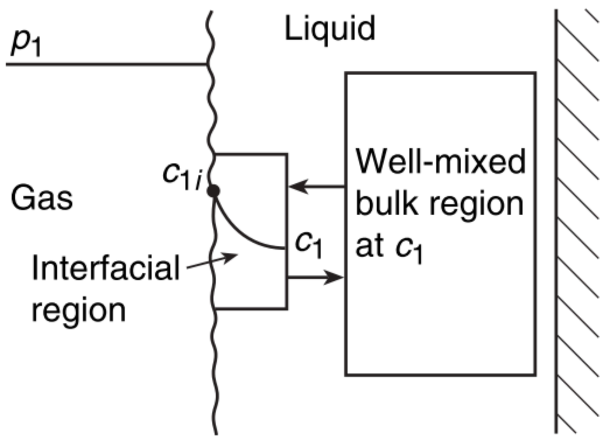
semi-infinite film diffusion




$$k = 2\sqrt{\frac{Dv_{max}}{\pi L}}$$

**Surface renewal theory**

Surface layer is renewed by bulk flow or turbulence




$$k = \sqrt{\frac{D}{\tau}}$$

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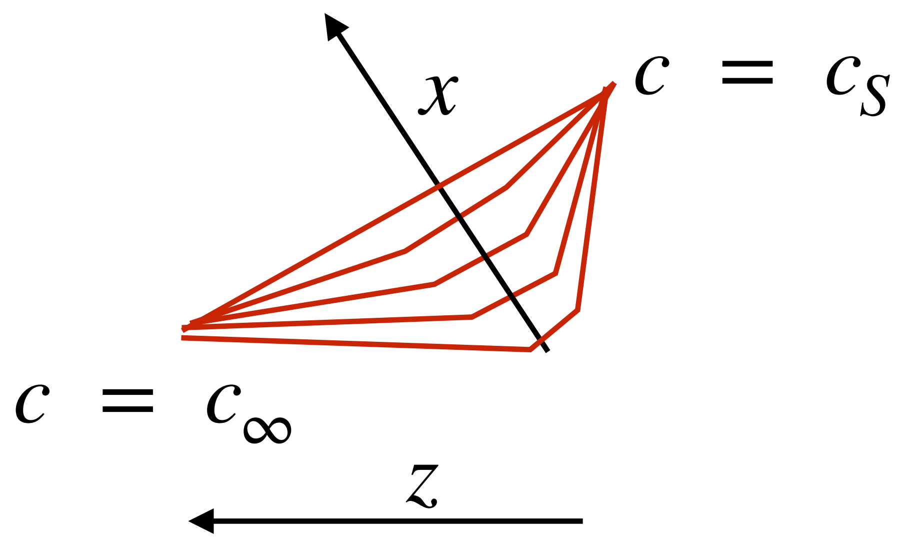
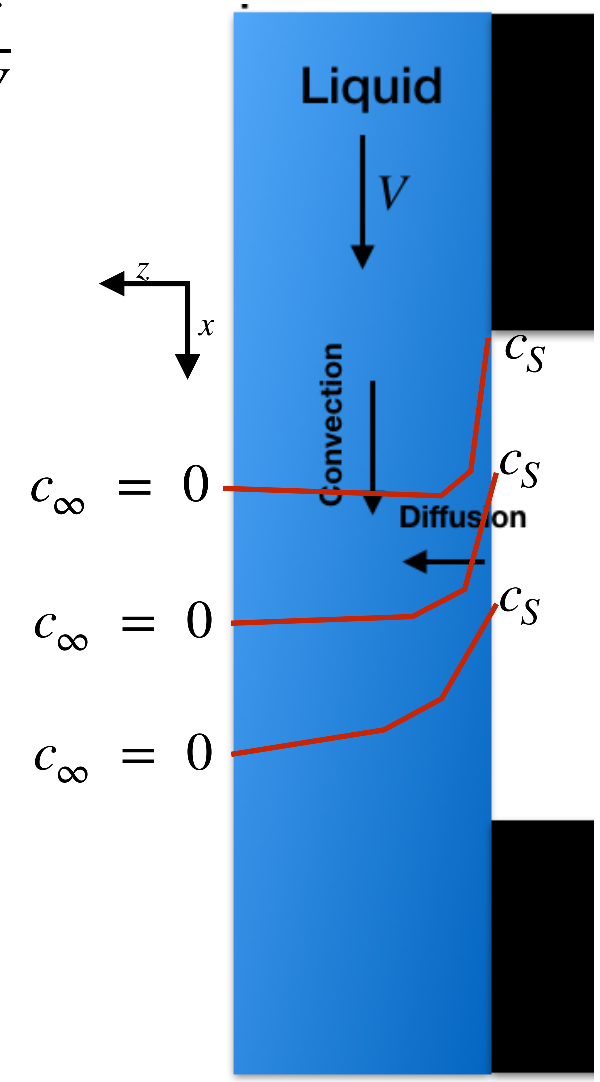
# Penetration theory: Gas diffusing in falling film

$$\frac{dc}{d\left(\frac{x}{V}\right)} = D\frac{d^2c}{dz^2} \qquad \frac{c(z,x) - c_S}{c_\infty - c_S} = erf\ \zeta \qquad \zeta = \frac{z}{\sqrt{4D\frac{x}{V}}}$$




**Can you calculate the flux at z = 0 ?**

**Can you derive** $k = 2\sqrt{\frac{Dv_{max}}{\pi L}}$

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# Calculation of flux

$$\frac{c(z,x)-c_S}{c_\infty - c_S} = erf\ \zeta = \frac{2}{\sqrt{\pi}}\int_0^\zeta \exp(-r^2)dr \qquad J = -D\frac{\partial c}{\partial z}$$

$$\frac{\partial c}{\partial z} = \left(\frac{dc}{d\zeta}\right)\left(\frac{\partial \zeta}{\partial z}\right)$$

$$\Rightarrow \frac{dc}{d\zeta} = (c_\infty - c_S)\frac{2}{\sqrt{\pi}} \exp\left(-\frac{z^2}{4D(x/V)}\right) \qquad x/V = t$$

$$\zeta = \frac{z}{\sqrt{4D\frac{x}{V}}} \qquad \Rightarrow \left(\frac{\partial \zeta}{\partial z}\right) = \frac{1}{\sqrt{4D(x/V)}}$$

$$\Rightarrow \frac{\partial c}{\partial z} = (c_\infty - c_S)\frac{1}{\sqrt{\pi D(x/V)}} \exp\left(-\frac{z^2}{4D(x/V)}\right)$$

$$\boxed{\Rightarrow J = -D\frac{\partial c}{\partial z} = -\sqrt{\frac{D}{\pi(x/V)}}\,(c_\infty - c_S)\exp\left(-\frac{z^2}{4D(x/V)}\right)}$$

$$J_{z=0} = \sqrt{\frac{D}{\pi(x/V)}}\,(c_s - c_\infty)$$

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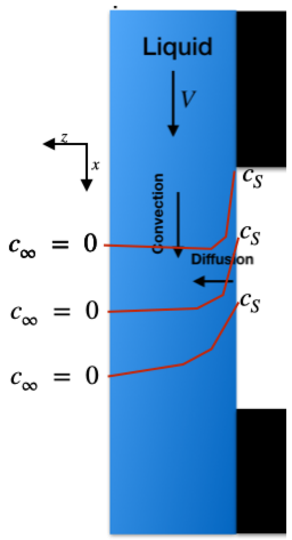
# Calculation of average flux




$$J_{z=0} = \sqrt{\frac{D}{\pi(x/V)}}\ (c_s - c_\infty)$$

**Integrating above over the entire length of the falling film, L**

$$J_{z=0,average} = \frac{1}{L}\int_0^L \sqrt{\frac{D}{\pi(x/V)}}\ (c_s - c_\infty)dx$$

$$J_{z=0,average} = \frac{2}{L}\sqrt{\frac{DLV}{\pi}}\ (c_s - c_\infty)$$

$$J_{z=0,average} = 2\sqrt{\frac{DV}{\pi L}}\ (c_s - c_\infty) \ = k(c_s - c_\infty)$$

$$k = 2\sqrt{\frac{DV}{\pi L}}$$

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# Film theory: approximate the thickness of interface layer

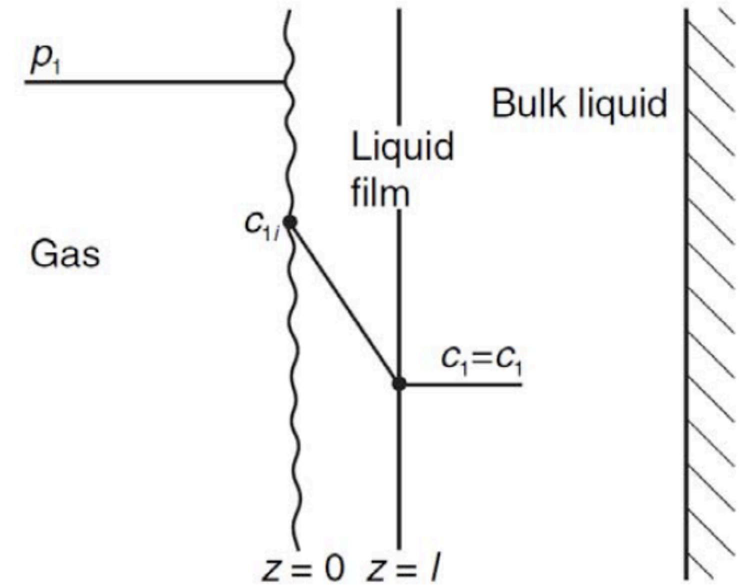
**$CO_2$ is being absorbed out of a gas using water flowing through a packed bed with following data**

- **$CO_2$ absorption rate is $2.3 \times 10^{-6}$ mol/cm$^2$sec at 27 °C.**
- **Partial pressure of $CO_2$ in gas is 10 atm.**
- **Henry's law coefficient is 600 atm.**
- **D for $CO_2$ in water is $1.9 \times 10^{-5}$ cm$^2$/sec.**
- **Assume bulk concentration of $CO_2$ in water to be zero.**

$$H = \frac{P_1}{x_{1i}}$$

$$N_1 = K_p(P_1 - Hx_1) = K_x\left(\frac{P_1}{H} - x_1\right)$$

$$\boxed{\frac{kl}{D} = 1}$$




$$\text{Absorption rate} = k(c_{1,i} - c_1) = kc_{1,i} = kcx_{1,i} = \frac{kcP_1}{H}$$

**Film theory:** $k = \frac{D}{l}$

$$\text{Absorption rate} = \frac{DcP_1}{Hl}$$

$$2.3 * 10^{-2} = \frac{1.9 * 10^{-9} * (55.5 * 10^3)}{l} * (10/600)$$

$$\Rightarrow l = 7.6 * 10^{-5} \text{ m} = 76\ \mu\text{m}$$

$$\Rightarrow k = 2.49 * 10^{-5} \text{ m s}^{-1}$$

**100 µm is the typical length-scale of interface**

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# Penetration theory: approximate the contact time, $L/v_{max}$

**Use the data from previous problem**

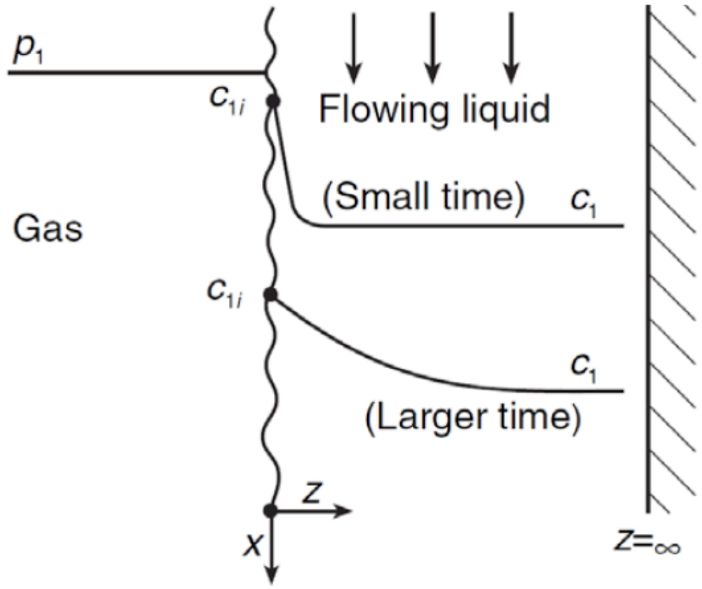
- **D for $CO_2$ in water is $1.9 \times 10^{-5}$ cm²/sec.**

$$k = 2.49 * 10^{-5} \text{ m s}^{-1}$$

**Penetration theory:**

$$k = 2 \cdot \sqrt{\frac{D \cdot v_{max}}{\pi \cdot L}}$$

$$L/v = 3.9 \text{ s}$$



# Surface renewal theory: approximate the surface renewal time

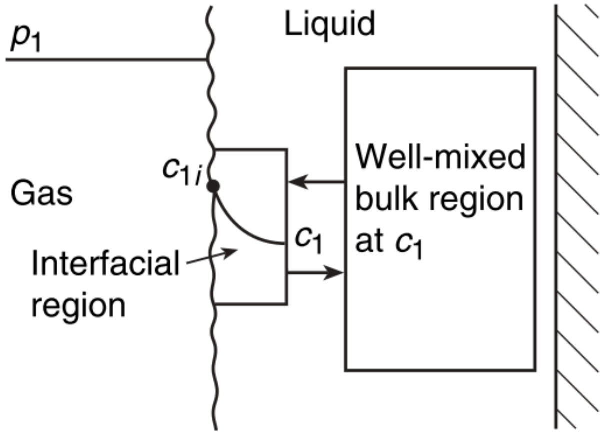
**Use the data from previous problem**

- **D for $CO_2$ in water is 1.9 x $10^{-5}$ $cm^2/sec$.**

$$k = 2.49 * 10^{-5} \text{ m s}^{-1}$$

**Surface renewal theory:**

$$k = \sqrt{\frac{D}{\tau}}$$




$$\tau = 3.06 \text{ s}$$

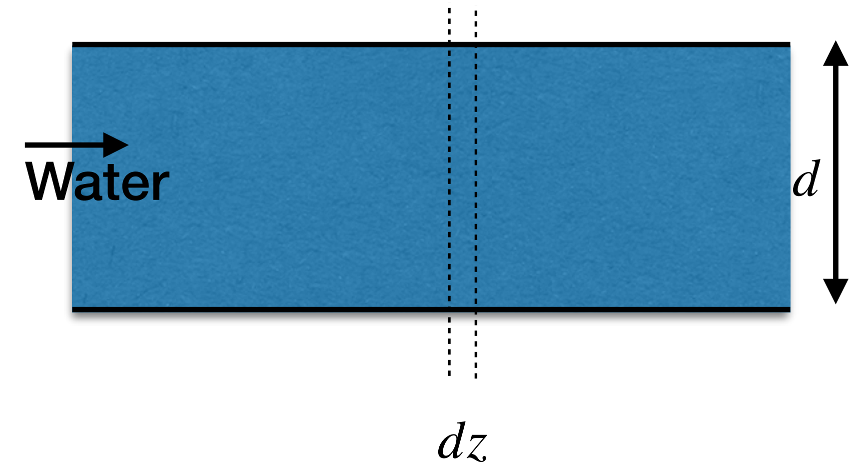
# In-class exercise problem

You are asked to estimate the mass transfer rate of benzoic acid in a process involving water flow inside a tube. The tube wall is made of solid benzoic acid. The saturated concentration of benzoic acid is $2.0 \times 10^{-2}$ g/cm$^3$. The water flow rate is 10 liter/min. Tube inner diameter is 1.0 cm. Tube length is 10 cm.

a) Is mass transfer between fluid-fluid interface or fluid-solid? (**Answer: Solid-Liquid**)

b) Calculate the mass transfer coefficient. Is it constant?

c) Derive an expression for the weight loss of benzoic acid.




$c_{1,sat} = 20 \text{ kg/m}^3 \qquad D = 10^{-5} \text{ cm}^2/\text{s} \qquad \nu = 10^{-6} \text{ m}^2/\text{s}$

$L = 0.1 \text{ m} \qquad d = 0.01 \text{ m}$

$$v = \frac{4Q}{\pi d^2} = 2.1 \text{ m/s} \qquad Re = \frac{dv}{\nu} = 21231$$

**Turbulent flow**

Turbulent flow through circular pipe $\quad \frac{kd}{D} = 0.026 \left(\frac{dv^0}{\nu}\right)^{0.8} \left(\frac{\nu}{D}\right)^{1/3} \quad$ $v^0$ = average velocity in slit, $d$ = pipe diameter

$$kd/D = 746 \qquad \Rightarrow k = 7.46 * 10^{-5} \text{ m/s}$$

**k is constant**

$$\text{Accumulation in water phase} = (\text{flow in} - \text{flow out}) + (\text{mass gained by mass transfer})$$

**Steady-state, accumulation = 0**

$$0 = (c_1 v) * A_1 |_z - (c_1 v) * A_1 |_{z+dz} + k A_2 (c_{1,i} - c_1)$$

$$A_1 = \pi d^2/4$$

$$A_2 = \pi d (dz)$$

$$\Rightarrow (c_1 v) * d^2/4 |_z - (c_1 v) * d^2/4 |_{z+dz} + k d (dz)(c_{1,i} - c_1) = 0$$

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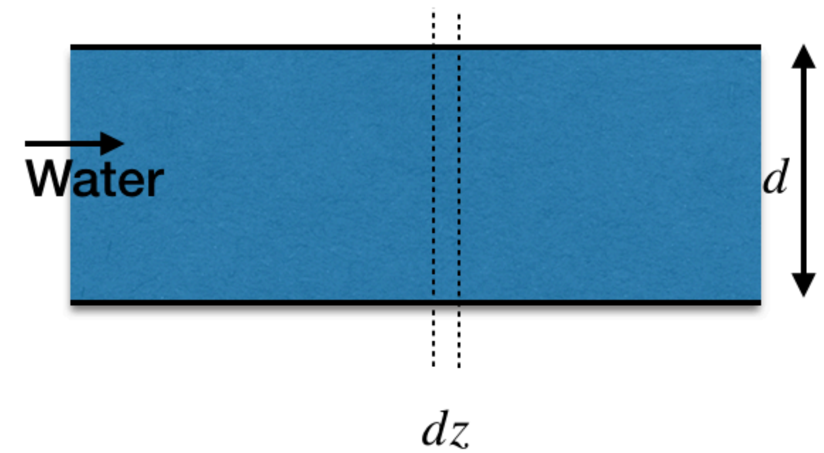
$$\Rightarrow (c_1 v) * d^2/4 \,|_z \; - \; (c_1 v) * d^2/4 \,|_{z+dz} \; + \; kd(dz)(c_{1,i} - c_1) = 0$$

$$\Rightarrow -\frac{(c_1)\,|_z \; - \; (c_1)\,|_{z+dz}}{dz} \; = \; \frac{k(c_{1,i} - c_1)}{(d/4)v}$$

**Similar solution to the previous problem (also similar boundary condition)** $c_1 \; = \; 0 \text{ when } z = 0,$

$$\Rightarrow c_1 \; = \; c_{1,i}\left[1 - \exp\left(-\frac{4kz}{vd}\right)\right] \; = \; c_{1,i} - c_{1,i}\exp\left(-\frac{4kz}{vd}\right)$$




$$\text{weight loss} \; = \; \int \mathrm{J}\pi\mathrm{d}\;\mathrm{dz}$$

$$\Rightarrow \text{weight loss} \; = \; \int \mathrm{k}(\mathrm{c}_{1,\mathrm{i}} - \mathrm{c}_1)\pi\mathrm{d}\;\mathrm{dz}$$

$$\Rightarrow \text{weight loss} \; = \; \pi\mathrm{kdc}_{1,\mathrm{i}}\int_0^{\mathrm{L}} \exp\left(-\frac{4\mathrm{kz}}{\mathrm{vd}}\right)\mathrm{dz}$$

$$\Rightarrow \text{weight loss} \; = \; \pi\mathrm{kdc}_{1,\mathrm{i}}\left[-\frac{\mathrm{vd}}{4\mathrm{k}}\exp\left(-\frac{4\mathrm{kz}}{\mathrm{vd}}\right)\right]\Big|_0^{\mathrm{L}} \qquad \Rightarrow \text{weight loss} \; = \; \frac{\pi\mathrm{d}^2}{4}\mathrm{vc}_{1,\mathrm{i}}\left[1 - \exp\left(-\frac{4\mathrm{kL}}{\mathrm{vd}}\right)\right]$$

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# Exercise problem 1
# Scrubbing problem

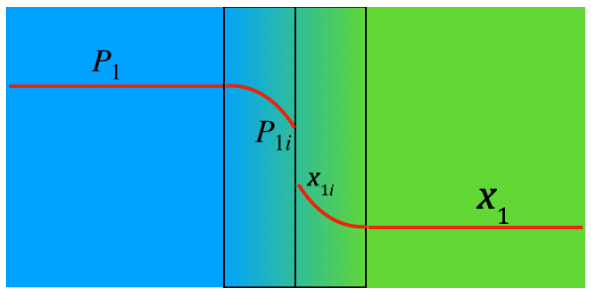
**Ammonia in a carrier gas (air) is being scrubbed from the gas phase to a solvent in a packed tower. The interaction (absorption) of $NH_3$ with the solvent is extremely strong and irreversible. Calculate the concentration of ammonia as a function of column height.**

$$x_{1i} = \bar{H}P_{1i}$$

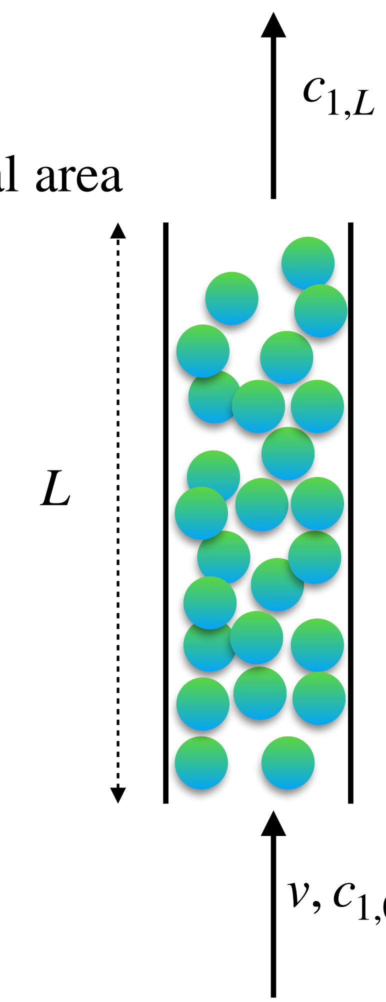
$$\Rightarrow \bar{H} \to \infty$$

$$\Rightarrow P_{1i} \to 0$$

$$\Rightarrow c_{1i} \to 0 \text{ (in gas phase)}$$




$$A = \text{cross} - \text{sectional area}$$

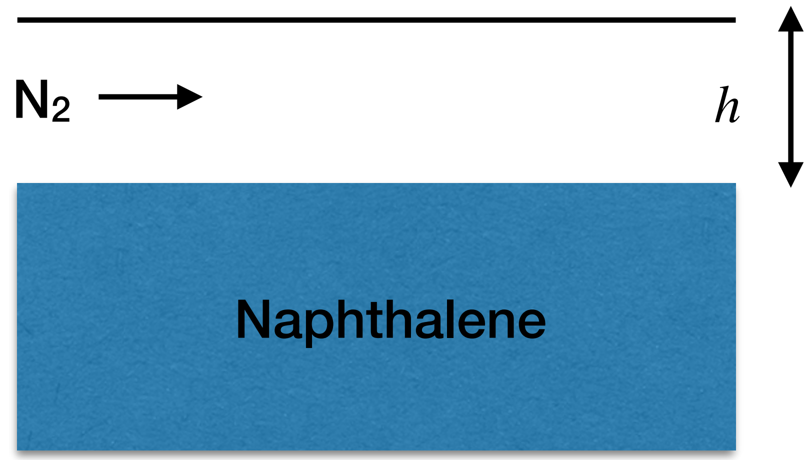


# Exercise problem 2

Nitrogen at 1 bar and 27 °C is enriched with naphthalene by flowing over a 1 meter long bed of solid naphthalene. If initially nitrogen is pure, and its velocity is 0.01 m/s, calculate

(a) the mass transfer coefficient of naphthalene in $N_2$.

(b) the partial pressure of naphthalene at the outlet given the width of bed is 10 cm and $h$ (height of nitrogen flow) = 10 cm.




$$p_{1,i} = p_{1,sat} = 0.1 \text{ mmHg}$$

$$\nu = 1.6 * 10^{-5} \text{ m}^2/\text{s} \qquad D = 0.1 \text{ cm}^2/\text{s}$$

# Solution to exercise problem 1

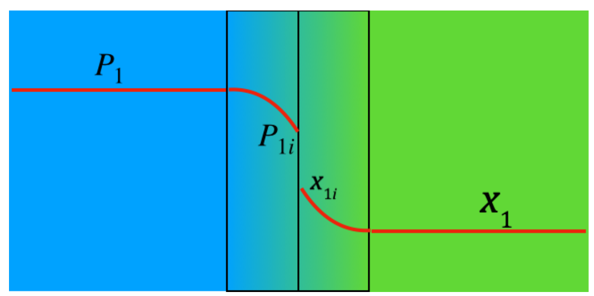
**Ammonia in a carrier gas (air) is being scrubbed from the gas phase to a solvent in a packed tower. The interaction (absorption) of $NH_3$ with the solvent is extremely strong and irreversible. Calculate the concentration of ammonia as a function of column height.**

$$x_{1i} = \bar{H} P_{1i}$$

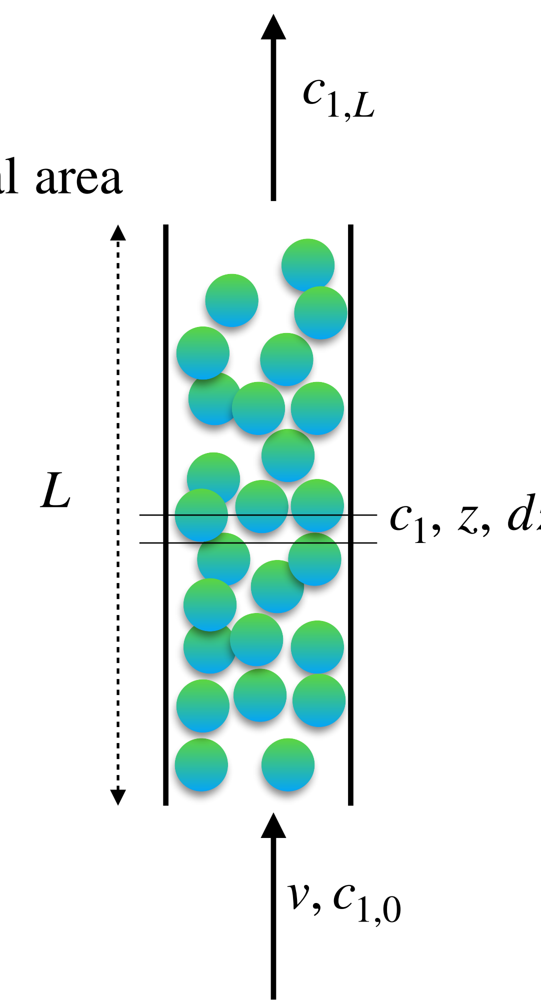
$$\Rightarrow \bar{H} \rightarrow \infty$$

$$\Rightarrow P_{1i} \rightarrow 0$$

$$\Rightarrow c_{1i} \rightarrow 0 \text{ (in gas phase)}$$




$$\text{A} = \text{cross} - \text{sectional area}$$




**Analysis on gas phase at height z**

$$\text{Accumulation in gas phase} = (\text{flow in} - \text{flow out}) - (\text{mass lost to absorption})$$

**Steady-state, accumulation = 0**

$$0 = (c_1 v) * A_1 |_z - (c_1 v) * A_1 |_{z+dz} - k A_2 (c_1 - c_{1,i})$$

$$A_1 = A$$

$$\Rightarrow (c_1 v) * A |_z - (c_1 v) * A |_{z+dz} - kaAdz(c_1 - 0)$$

$$A_2 = aAdz$$

$$\Rightarrow \frac{dc_1}{dz} = -\frac{kac_1}{v}$$

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Ammonia in a carrier gas (air) is being scrubbed from the gas phase to a solvent in a packed tower. The interaction (absorption) of $NH_3$ with the solvent is extremely strong and irreversible. Calculate the concentration of ammonia as a function of column height.

$$\Rightarrow \frac{dc_1}{dz} = -\frac{kac_1}{v}$$

$$\Rightarrow \ln c_1 = -\frac{kaz}{v} + \text{constant}$$

$c_1 = c_{1,0}$ when z = 0,

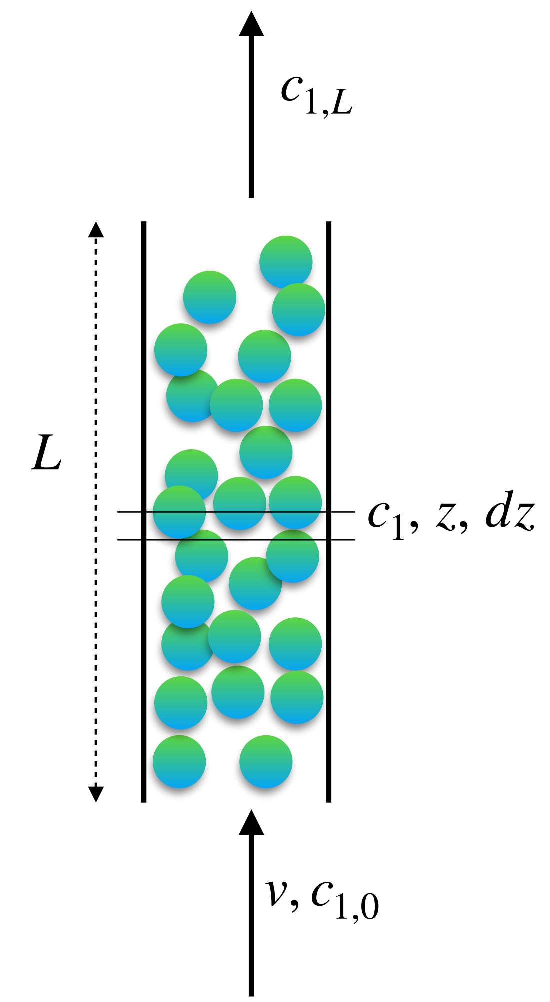
$$\Rightarrow \text{constant} = \ln \mathrm{c}_{1,0}$$

$$\Rightarrow \ln\left(\frac{c_1}{c_{1,0}}\right) = -\frac{kaz}{v}$$

$$\Rightarrow ka = -\frac{v}{z}\ln\left(\frac{c_1}{c_{10}}\right)$$

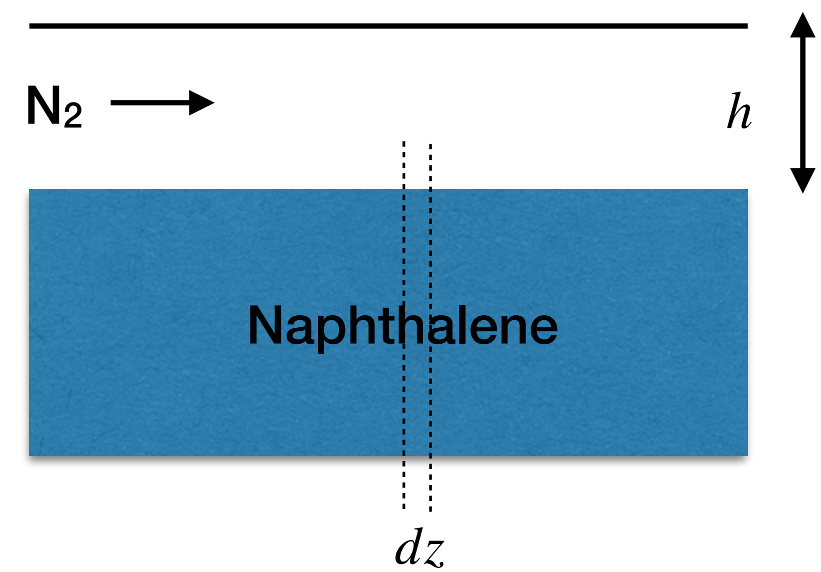
$$\Rightarrow c_1 = c_{1,0}\exp\left(-\frac{kaz}{v}\right)$$

$$\Rightarrow ka = -\frac{v}{L}\ln\left(\frac{c_{1L}}{c_{10}}\right)$$



# Solution to exercise problem 2

**Nitrogen at 1 bar and 27 °C is enriched with naphthalene by flowing over a 1 meter long bed of solid naphthalene. If initially nitrogen is pure, and its velocity is 0.01 m/s, calculate**
**(a) the mass transfer coefficient of naphthalene in $N_2$.**
**(b) the partial pressure of naphthalene at the outlet given the width of bed is 10 cm and H (height of nitrogen flow) = 10 cm.**




$p_{1,i} = p_{1,sat} = 0.1 \text{ mmHg}$

$\nu = 1.6 * 10^{-5} \text{ m}^2/\text{s}$ $\qquad D = 0.1 \text{ cm}^2/\text{s}$

$v = 0.01 \text{ m/s}$ $\qquad L = 1 \text{ m}$ $\qquad Re = \frac{lv}{\nu} = 625$ $\qquad$ **Laminar flow**

Laminar flow along flat plate[c] $\qquad \frac{kL}{D} = 0.646 \left(\frac{Lv^0}{\nu}\right)^{1/3} \left(\frac{\nu}{D}\right)^{1/3}$ $\qquad L$ = plate length, $v^0$ = bulk velocity

$kL/D = 6.46$ $\qquad \Rightarrow k = 6.46 * 10^{-5} \text{ m/s}$ $\qquad$ This is an average k over entire length

Accumulation in gas phase = (flow in − flow out) + (mass gained by mass transfer)

**Steady-state, accumulation = 0**

$$0 = (c_1 v) * A_1 |_z - (c_1 v) * A_1 |_{z+dz} + kA_2(c_{1,i} - c_1)$$

$$A_1 = Wh$$

$$A_2 = Wdz$$

$$\Rightarrow (c_1 v) * h |_z - (c_1 v) * h |_{z+dz} + kdz(c_{1,i} - c_1) = 0$$

EPFL

$$\Rightarrow (c_1 v) * h\,|_z \;-\; (c_1 v) * h\,|_{z+dz} \;+\; k dz (c_{1,i} - c_1) = 0$$

$$\Rightarrow -\frac{(c_1)\,|_z \;-\; (c_1)\,|_{z+dz}}{dz} \;=\; \frac{k(c_{1,i} - c_1)}{hv}$$

$$\Rightarrow \frac{dc_1}{dz} \;=\; \frac{k(c_{1,i} - c_1)}{hv}$$

$$\Rightarrow -\ln(c_{1,i} - c_1) \;=\; \frac{kz}{hv} \;+\; \text{constant}$$

$$c_1 \;=\; 0 \text{ when z} = 0,$$

$$\Rightarrow \text{constant} \;= -\ln(\mathrm{c}_{1,\mathrm{i}})$$

$$\Rightarrow -\ln\left(\frac{c_{1,i} - c_1}{c_{1,i}}\right) \;=\; \frac{kz}{hv}$$

$$\Rightarrow \frac{c_{1,i} - c_1}{c_{1,i}} \;=\; \exp\left(-\frac{kz}{hv}\right)$$

$$\Rightarrow c_{1,i} - c_1 \;=\; c_{1,i} \exp\left(-\frac{kz}{hv}\right)$$

$$\Rightarrow c_1 \;=\; c_{1,i}\left[1 - \exp\left(-\frac{kz}{hv}\right)\right]$$

**Check the solution**

$$z = 0, \qquad c_1 = 0$$

$$z = L \qquad c_1 \;=\; c_{1,i}\left[1 - \exp\left(-\frac{kL}{hv}\right)\right]$$

$$c_1 \;=\; 0.06 c_{1,i}$$

$$p_1 = 0.06 p_{1,i} = 0.006 \text{ mmHg}$$

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